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REVIEW PAPER

Reduced-order modeling for engineering systems: survey and opportunities for digital twins

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Abstract
Digital twins have been the subject of significant and growing interest across scientific, engineering, and medical domains. And yet the promise of digital twins relies critically on the availability of efficient and accurate computational models, as all of the computations required for and from digital twins are outer-loop/many-query analyses: digital twin synchronization based on data uses iterative algorithms or ensemble methods for inference and estimation; real-time decision-making with digital twins is based on many-query computations for optimization and control, sometimes under uncertainty, which exacerbates the many-query nature of the computation. These computations must happen in or near real time to be useful. However, many computational models for engineering systems arise from the discretization of governing partial differential equations on high-dimensional computational meshes, making the resulting models expensive to both store and evaluate. To realize the promise of digital twins, inexpensive and fit-for-purpose *reduced-order models* offer much promise, due to their success across many areas of engineering over the past half-century. The goal of this survey is to provide the engineering community with an overview of the state of the art in reduced-order modeling with an eye towards promise for digital twin applications. We will survey state approximation methods, model reduction based on dynamical systems and control theory, as well as goal-oriented model reduction methods accounting for a variety of additional demands, such as structure preservation, suitability for inverse problems, stability, and others.

Keywords Digital twins · Model reduction · Outer-loop applications · Approximations

1 Introduction

Computational models that enable predictive simulation have transformed the design of multidisciplinary engineering systems by reducing the need for costly and time-consuming experiments. In particular, *outer-loop* design analyses—which are analyses such as optimization or uncertainty quantification where system performance must be evaluated many times at different parameters or inputs—are often cost prohibitive when performance evaluation relies solely on experiments. Affordable computational models can make outer-loop analyses tractable, leading to more robust and efficient designs. These models can also help in real-time decision making.

However, barriers to such outer-loop applications remain in many practical engineering settings due to the complementary challenges of (i) the high computational cost of accurate models for complex and multidisciplinary systems and (ii) the need for a large number of model evaluations. These challenges are particularly salient in the development of *digital twins*, a term which describes a virtual representation (e.g., a suite of models) of a specific instance of a physical engineering system, which is (a) synchronized in real time based on data from the physical system, and (b) can be used for real-time outer-loop analysis to support operational decision-making related to, e.g., control, system maintenance, and evaluation of 'what if' scenarios.

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Digital twins have been the subject of significant and growing interest across scientific, engineering, and medical domains (NASEM 2024). In medicine, digital twins promise the advent of personalized medicine. In engineering, digital twins have the potential to enable the efficient operation and maintenance of fleets of vehicles, production plants, highly complex machines, and many more. And yet the promise of digital twins relies critically on the availability of cheap, compact computational models. All of the computations required for and from digital twins are outer-loop/many-query analyses: physical-to-digital (P2D) synchronization is based on data use algorithms for inference and estimation, which inevitably are iterative or ensemble methods requiring many queries of the model, and digital-to-physical (D2P) decision making is based on many-query computations for optimization and control, sometimes under uncertainty, which exacerbates the many-query nature of the computation. These computations must happen in (near) real time to be useful, and in many cases must happen on fairly lightweight computers (e.g., on board a drone or on a standard laptop); if every digital twin required supercomputing resources, it would be hard to realize the potential benefits of digital twins for many of the envisioned applications. Since many computational models for engineering systems arise from the discretization of governing partial differential equations (PDEs) on high-dimensional computational meshes, this makes the resulting models expensive to both store and evaluate, making them ill-suited for use in digital twins. To realize the promise of digital twins, inexpensive and compact *reduced-order models (ROMs)* offer much promise, due to their success across many areas of engineering over the past half-century.

The goal of this survey is to provide the engineering community with an overview of the state of the art in reduced-order modeling with an eye towards promise for digital twin applications—the focus of this special issue collection.[1] The terms 'reduced model,' 'reduced-order model,' and 'model reduction' have been circulating in different communities for decades with various meanings. In some cases, the term 'reduced(-order) model' is used to describe a model of simplified physics (neglecting higher-order effects, physical or chemical processes, detailed mechanisms, etc.). For example, in chemical kinetics for combustion, model reduction refers to reducing the number of elementary reactions and chemical species needed to simulate combustion processes (Maas and Pope 1992). In plasma physics, the magnetohydrodynamic equations are referred to as a reduced(-order) model of the fully kinetic description for the particle distribution function. In this paper, we use the definition of reduced(-order) model more common to the computational mathematics community, which specifically describes a class of methods that approximate the high-dimensional governing equations of a system in a low-dimensional latent space, yielding equations said to be a reduced-order model that can be more cheaply evaluated yet provides acceptable levels of accuracy. In contrast, the original high-dimensional governing equations are said to define the *full-order model (FOM)*.

The field of reduced-order modeling has developed a rich set of tools and theories to address a wide variety of static and dynamical systems, given different goals on desired outcomes. It is not the intent of this survey to provide a historical account of developments in reduced-order modeling, neither will we be able to do justice to every contribution during five successful decades of model reduction research. Rather, we seek here to provide an overview of the main distinct paradigms that have arisen in model reduction methods, and to highlight model reduction successes in engineering application as well as challenges and opportunities that remain for digital twins. Interested readers are referred to several excellent books and survey papers (Freund 2003; Rozza et al. 2008; Quarteroni et al. 2011; Benner et al. 2015; Hesthaven et al. 2016; Antoulas et al. 2020; Hesthaven et al. 2022b) for more in-depth technical discussion of model reduction.

The most general and widely used model reduction approaches are what we refer to in this work as 'state approximation' approaches. These approaches, discussed in Sect. 2, use system state data to identify a low-dimensional subspace in which the available data can be well approximated. Once this low-dimensional subspace has been identified, the ROM can be obtained either via intrusive methods which project the operators of the high-dimensional governing equations onto the low-dimensional subspace, or via non-intrusive methods which fit reduced operators to data. We emphasize that the intrusive ROMs are *physics-based* models that directly solve the underlying governing equations in the approximation space or on a manifold, in contrast to physics-agnostic surrogate modeling approaches such as Gaussian processes or neural networks. Figure 1 illustrates the main ideas of the state approximation model reduction approach. State-approximation approaches are highly flexible and can apply to a variety of systems with different structures, including static and dynamic systems, as well as both linear and nonlinear systems.

Other model reduction approaches are more narrowly applicable and take advantage of specific structure of the system of interest. In particular, dynamical systems- and control-theoretic model reduction, covered in Sect. 3, is rooted in the systems and control theory community, where early reduction techniques were developed in the late 1970s and 1980s. Those methods largely focus on accurately capturing the input–output behavior of the complex system, characterized through the system transfer function in the lin-

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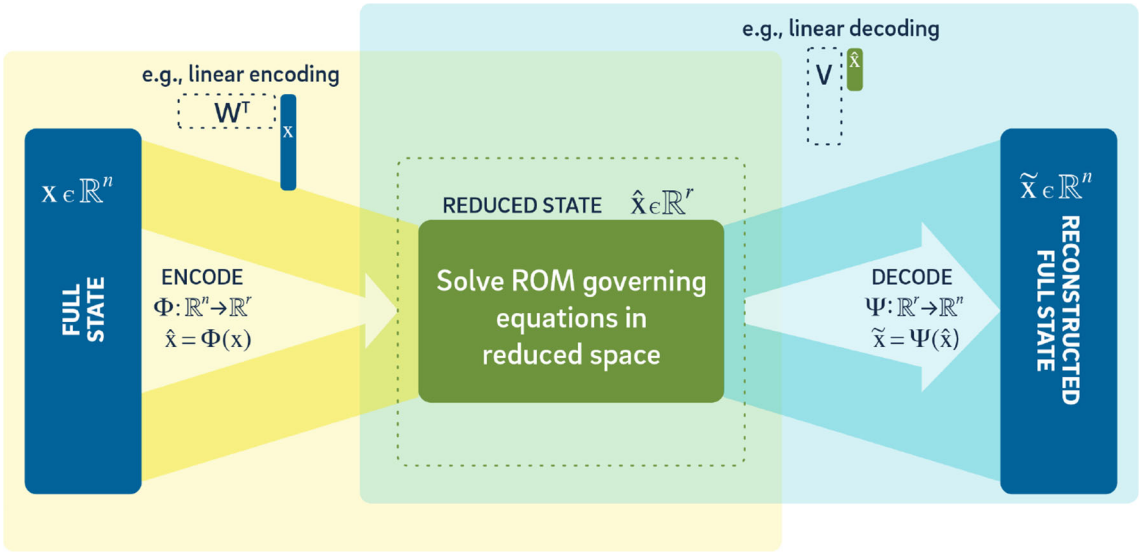

Fig. 1 Schematic of the three main components of state approximation model reduction methods: (1) encoding the high-dimensional state into a low-dimensional reduced space, (2) modeling the system in the reduced space, (3) decoding the reduced solution to the original high-dimensional variables. We emphasize that for classical ROMs, the ROM governing equations arise from enforcing the original physics equations in the reduced space

ear time-invariant case, and use additional system invariants in their objective, such as the Hankel singular values. While originally developed assuming full knowledge of the system's mathematical model (intrusive), many developments over the past decade provided extensions to the non-intrusive case.

Table 1 provides a brief categorization, albeit with soft boundaries, of the ROM methods surveyed in Sects. 2 and 3. Beyond the large family of system-theoretic approaches, we discuss *goal-oriented* approaches that exploit other types of problem structure in Sect. 4. These approaches include methods that seek to preserve physical principles such as energy conservation, as well as methods that tailor the ROM to particular outer-loop analyses of interest, including inference and optimization. Section 5 highlights successful uses of model reduction methods for large-scale engineering applications. Section 6 discusses available software and benchmark problems. Section 7 contains concluding remarks with an eye towards opportunities for digital twins.

2 State-approximation methods

We begin by surveying a large class of model reduction methods that we term 'state approximation' methods because these methods aim for accurate predictions of the system state. This contrasts with the system-theoretic methods discussed next in Sect. 3, which aim for accurate approximations of the input–output map, and with goal-oriented methods (Sect. 4) which target goals other than state approximation. We begin by introducing the main ideas behind proper orthogonal decomposition (POD) and reduced basis (RB) methods, the two primary families of state approximation model reduction methods, in Sect. 2.1. Section 2.2 then discusses methods for addressing computational challenges arising in model reduction of nonlinear problems. Section 2.3 then presents an overview of non-intrusive, data-driven ROM methods.

2.1 Proper orthogonal decomposition and reduced basis methods

This section introduces two related foundational model reduction methods for state approximation: (i) proper orthogonal decomposition (POD) methods and (ii) reduced basis (RB) methods. Such methods are applicable in fairly general settings, which we describe in Sect. 2.1.1. While POD and RB methods have different historical origins, both families are largely based on the same principle, which is to use system state data to define a low-dimensional space in which to approximate the system state, which we describe in Sect. 2.1.2. This low-dimensional space is then used to define a reduced-order model, most commonly by Galerkin projection, as described in Sect. 2.1.3. Section 2.1.4 discusses various strategies for selecting the state data which defines the basis. We conclude this section with some discussion on differences between the POD and RB literature in Sect. 2.1.5.

Table 1 Intrusive model reduction methods require the ability to manipulate the high-fidelity computer code to implement projections and simulate the ROM. Non-intrusive methods do not require such access and learn the ROM by mimicking intrusive MOR steps

	Intrusive methods	Non-intrusive methods
State approximation	Proper orthogonal decomposition (POD)	Dynamic mode decomposition (DMD)
	Reduced basis (RB) method	Operator inference (OpInf)
		Machine learning approaches
System theoretic	Interpolatory $\mathcal{H}_2, \mathcal{H}_\infty$ model reduction	Loewner method
	Moment matching	Vector fitting
	Balanced truncation	AAA algorithm
		Eigensystem realization algorithm (ERA)

2.1.1 Setting

The power of state approximation methods is that they are quite general. In this section we will consider any of the following types of systems:

$$\begin{aligned} \text{linear static system} \quad & \mathbf{A}(\boldsymbol{\mu})\mathbf{x} = \mathbf{b}(\boldsymbol{\mu}), \\ \text{nonlinear static system} \quad & \mathbf{f}(\mathbf{x}; \boldsymbol{\mu}) = \mathbf{0}, \\ \text{linear dynamical system} \quad & \dot{\mathbf{x}} = \mathbf{A}(\boldsymbol{\mu})\mathbf{x}, \\ \text{nonlinear dynamical system} \quad & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}; \boldsymbol{\mu}), \end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the system state, $\boldsymbol{\mu} \in \mathcal{D}$ is a parameter with domain $\mathcal{D}$, $\mathbf{A}(\boldsymbol{\mu}) \in \mathbb{R}^{n\times n}$ is a parameter-dependent matrix operator, $\mathbf{b}(\boldsymbol{\mu})$ is a parameter-dependent forcing vector, and $\mathbf{f} : \mathbb{R}^n \times \mathcal{D} \to \mathbb{R}^n$ is a parameter-dependent nonlinear operator. State-approximation methods for model reduction may generally be applied to any of the above types of systems, although the availability of accuracy and/or computational efficiency guarantees will depend on certain types of structure that we will discuss. It is also possible to add control terms to the dynamical systems, similar to Sect. 3, but for sake of simplicity we will not consider them here.

2.1.2 Definition of approximation basis from snapshot data

Let $\mathbf{X} \in \mathbb{R}^{n\times K}$ denote a matrix whose K columns consist of state data $\{\mathbf{x}_k\}_{k=1}^K$ from the system of interest (1). These data consist of 'snapshots' of the state at different times, for different parameters, for varying initial conditions, or some combination of these. Let $\mathbf{X} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top$ denote the singular value decomposition (SVD) of $\mathbf{X}$, where $\mathbf{U} \in \mathbb{R}^{n\times n}$, $\boldsymbol{\Sigma} \in \mathbb{R}^{n\times K}$, and $\mathbf{V} \in \mathbb{R}^{K\times K}$. Denote by $\mathbf{U}_r \in \mathbb{R}^{n\times r}$ the submatrix of $\mathbf{U}$ consisting of the r leading columns of $\mathbf{U}$. We call $\mathbf{U}_r$ a POD basis matrix of dimension r (Berkooz et al. 1993; Sirovich and Sirovich 1989). Note that because $\mathbf{U}$ has orthonormal columns, $\mathbf{U}_r\mathbf{U}_r^\top$ defines an orthogonal projection matrix whose range is the span of the columns of $\mathbf{U}_r$. The Eckart–Young theorem tells us that the columns of $\mathbf{U}_r$ are an *optimal basis* of rank r for reconstructing the snapshot matrix $\mathbf{X}$, i.e., (Trefethen and Bau 1997; Demmel 1997),

$$\begin{aligned} &\|\mathbf{U}_r\mathbf{U}_r^\top\mathbf{X} - \mathbf{X}\|_F^2 \le \|\mathbf{P}_r\mathbf{X} - \mathbf{X}\|_F^2 \\ &\text{for any projector } \mathbf{P}_r \text{ with } \operatorname{rank}(\mathbf{P}_r) \le r. \end{aligned} \tag{2}$$

The POD basis vectors contained in $\mathbf{U}_r$ can be equivalently defined as the leading eigenvectors of the snapshot covariance matrix $\mathbf{X}\mathbf{X}^\top$; that procedure initially led to the term 'proper orthogonal decomposition' within the context of modal analysis of statistically stationary turbulent flows (Sirovich and Sirovich 1989; Berkooz et al. 1993). Note that the cost of computing the POD basis via the economy-sized SVD is $\mathcal{O}(n^2K)$, and the cost of computing the POD basis via eigendecomposition of the covariance matrix is $\mathcal{O}(n^3)$. When n is large, this may be too expensive. The 'method of snapshots' is a more computationally efficient way to compute the POD basis when $K \ll n$ (Sirovich 1987). This defines the POD basis as $\mathbf{U}_r = \mathbf{X}\mathbf{V}_r\boldsymbol{\Lambda}_r^{-\frac{1}{2}}$, where $\mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^\top = \mathbf{X}^\top\mathbf{X}$ is the eigendecomposition of $\mathbf{X}^\top\mathbf{X} \in \mathbb{R}^{K\times K}$, and $\mathbf{V}_r \in \mathbb{R}^{K\times r}$ and $\boldsymbol{\Lambda}_r \in \mathbb{R}^{r\times r}$ are the left and upper-left submatrices of $\mathbf{V}$ and $\boldsymbol{\Lambda}$, respectively. The POD decomposition is known by several other names in other disciplines, including principal component analysis (Jolliffe 2011) and the Karhunen–Loéve expansion (Karhunen 1947; Loève 1977). We discuss several different strategies for choosing the data used to define the POD basis in Sect. 2.1.4.

2.1.3 Reduced-order model definition via Galerkin projection

The most common way to obtain a ROM using the POD basis is via Galerkin projection (Benner et al. 2015; Ghattas and Willcox 2021). That is, one first approximates the state as $\mathbf{x} \approx \mathbf{U}_r\hat{\mathbf{x}}$, where $\hat{\mathbf{x}} \in \mathbb{R}^r$ is called the *reduced* state. Then, we substitute this approximation into the original governing equation (1) and enforce the Galerkin orthogonality condition that the approximation residual be orthogonal to Range($\mathbf{U}_r$). For simplicity, we first present the resulting approximations to the dynamical systems in (1) for the spe-

cial case with no parameter dependence:

$$\text{linear ROM} \qquad \dot{\hat{\mathbf{x}}} = \mathbf{U}_r^\top \mathbf{A}\mathbf{U}_r\hat{\mathbf{x}} \equiv \hat{\mathbf{A}}\hat{\mathbf{x}}, \tag{3}$$

$$\text{nonlinear ROM} \qquad \dot{\mathbf{x}} = \mathbf{U}_r^\top \mathbf{f}(\mathbf{U}_r\hat{\mathbf{x}}), \tag{4}$$

where we have defined the reduced operator $\hat{\mathbf{A}} := \mathbf{U}_r^\top \mathbf{A}\mathbf{U}_r \in \mathbb{R}^{r\times r}$. For linear systems, the reduced operator $\hat{\mathbf{A}}$ must be computed only once in the offline phase, and can then be stored so that the ROM can then be used for efficient computation in the online phase, e.g., for many different initial conditions (Rozza et al. 2008; Benner et al. 2015). This type of offline/offline decomposition is also possible for nonlinear terms that are polynomial in the reduced state (Kramer and Willcox 2019; Benner and Breiten 2015). However, for general nonlinearities, the nonlinear ROM in (4) requires evaluation of the high-dimensional function $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$. This is referred to as the computational bottleneck of nonlinear model reduction, discussed in more detail in Sect. 2.2.

We emphasize that the ROM governing equations are the *true physical governing equations* of the underlying system, projected onto the reduced space. We note, however, that in some cases the POD-Galerkin ROM may not be stable even if the underlying FOM is stable. This has motivated the development of a class of POD-based approaches called least-squares Petrov–Galerkin (LSPG) methods, which define the ROM *implicitly* through the minimization of a discrete-time residual (Carlberg et al. 2011, 2017; Parish et al. 2020), and have been numerically demonstrated to exhibit improved stability properties. We also discuss additional ROM stabilization approaches for state approximation methods further in Sect. 4.3.

We now consider Galerkin projection of *parametrized* systems, yielding the following ROM forms:

$$\begin{aligned}
\text{linear static ROM} \qquad & \mathbf{U}_r^\top \mathbf{A}(\boldsymbol{\mu})\mathbf{U}_r\hat{\mathbf{x}} = \mathbf{U}_r^\top \mathbf{b}(\boldsymbol{\mu}),\\
\text{nonlinear static ROM} \qquad & \mathbf{U}_r^\top \mathbf{f}(\mathbf{U}_r\hat{\mathbf{x}};\boldsymbol{\mu}) = \mathbf{0},\\
\text{linear ROM} \qquad & \dot{\hat{\mathbf{x}}} = \mathbf{U}_r^\top \mathbf{A}(\boldsymbol{\mu})\mathbf{U}_r\hat{\mathbf{x}},\\
\text{nonlinear ROM} \qquad & \dot{\mathbf{x}} = \mathbf{U}_r^\top \mathbf{f}(\mathbf{U}_r\hat{\mathbf{x}};\boldsymbol{\mu}).
\end{aligned} \tag{5}$$

As before, nonlinear model reduction suffers from the computational bottleneck of needing to evaluate $\mathbf{f}$. However, introducing parameter dependence now complicates the linear case as well: one can define reduced-order matrices $\hat{\mathbf{A}}(\boldsymbol{\mu}) := \mathbf{U}_r^\top \mathbf{A}(\boldsymbol{\mu})\mathbf{U}_r$ and $\hat{\mathbf{b}}(\boldsymbol{\mu}) := \mathbf{U}_r^\top \mathbf{b}(\boldsymbol{\mu})$, but these reduced operators are parameter-dependent. Usually, one wishes to be able to efficiently evaluate the ROMs at many different parameters during the online phase. The parameter dependence of the reduced operators prevents us from being able to pre-compute the reduced operators offline and simply use the reduced operators for online evaluation. Instead, one common strategy is to choose a fixed set of training parameters $\{\boldsymbol{\mu}_i\}_{i=1}^M$, and to pre-compute reduced operators $\{\hat{\mathbf{A}}(\boldsymbol{\mu}_i)\}_{i=1}^M$ during the offline phase. During the online phase, one can then approximate $\hat{\mathbf{A}}(\boldsymbol{\mu})$ by interpolating the between the nearest pre-computed reduced operators (Bui-Thanh et al. 2003; Amsallem and Farhat 2008; Degroote et al. 2010; Panzer et al. 2010). Another strategy that is common in the reduced basis literature exploits affine structure in the parameter dependence of the reduced operators (Prud'homme et al. 2002; Rozza et al. 2008; Quarteroni et al. 2011; Hesthaven et al. 2022b). That is, suppose the full-order operators can be written as a linear combination of parameter-independent operators with parameter-dependent weights:

$$\mathbf{A}(\boldsymbol{\mu}) = \sum_{q=1}^{Q_A} \theta_q^A(\boldsymbol{\mu})\mathbf{A}_q, \qquad \mathbf{b}(\boldsymbol{\mu}) = \sum_{q=1}^{Q_b} \theta_q^b(\boldsymbol{\mu})\mathbf{b}_q, \tag{6}$$

where $\theta_q^A, \theta_q^b : \mathcal{D} \to \mathbb{R}$. In this case, the reduced operators may be written as

$$\hat{\mathbf{A}}(\boldsymbol{\mu}) = \sum_{q=1}^{Q_A} \theta_q^A(\boldsymbol{\mu})\hat{\mathbf{A}}_q, \qquad \hat{\mathbf{b}}(\boldsymbol{\mu}) = \sum_{q=1}^{Q_b} \theta_q^b(\boldsymbol{\mu})\hat{\mathbf{b}}_q, \tag{7}$$

where $\hat{\mathbf{A}}_q = \mathbf{U}_r^\top \mathbf{A}_q \mathbf{U}_r \in \mathbb{R}^{r\times r}$ for $q = 1, 2, \ldots, Q_A$ and $\hat{\mathbf{b}}_q = \mathbf{U}_r^\top \mathbf{b}_q \in \mathbb{R}^r$ for $q = 1, 2, \ldots, Q_b$. The parameter-independent reduced operators $\hat{\mathbf{A}}_q$ and $\hat{\mathbf{b}}_q$ can then be pre-computed and stored during the offline phase. Then, during the online phase, they can be used to assemble the parameter-dependent operators via (7). Note that for this online computation to be efficient, we require that the number of terms in the sum be moderate, i.e., $Q_A, Q_b \ll n$, and that the functions defining the parameter-dependent weights θ_q^A and θ_q^B be cheap to evaluate. Parameter-dependent operators not of the form (6) may still be approximated as such via the empirical interpolation method (Barrault et al. 2004); extensions of this method are now also popularly used for hyperreduction of nonlinear ROMs (see Sect. 2.2).

2.1.4 Remarks on snapshot data selection

We note that the Eckart–Young result (2) concerns optimal reconstruction only of the available snapshot data in $\mathbf{X}$. The selection of snapshots therefore has critical influence on the ability of the POD basis to accurately approximate new, unseen states (Ghattas and Willcox 2021). In POD's origins in the analysis of turbulent flows, the snapshots contained in $\mathbf{X}$ were expected to come from statistically stationary flows (Lumley 1967; Berkooz et al. 1993), such that the snapshot covariance matrix $\mathbf{X}\mathbf{X}^\top$ is a good estimate of the true state covariance, and the POD basis vectors are therefore optimal for reconstructing any state drawn from the stationary

distribution. Similarly, when parameters or initial conditions of the system of interest are viewed as random variables, the snapshots in $\mathbf{X}$ may be collected from sampling states and trajectories associated with samples of the random parameters and initial conditions. With enough samples, the POD basis error in reconstructing the $\mathbf{X}$ is representative of the error in reconstructing unseen data. However, POD methods are often used in other contexts, for example, one common approach is to collect snapshot data at time steps less than a certain time T, and then to use the resulting basis to define a ROM for extrapolation for $t > T$ (Qian et al. 2022; McQuarrie et al. 2021). Another approach for systems with control inputs is to generate snapshots from either the unforced system or by using a reference control input (Ravindran 2000; Gubisch and Volkwein 2017).

For parametrized problems, a common strategy is to collect snapshot data that are globally representative of the parameter domain. This may be done by collecting snapshots at a grid (or other space-filling design) of parameter values within the parameter domain $\mathcal{D}$ (Peherstorfer and Willcox 2016a; Ghattas and Willcox 2021; Rozza et al. 2008). With enough data, this strategy will lead to accurate results, but the cost of gathering sufficient snapshots for accuracy can be expensive, particularly when the dimension of $\mathcal{D}$ is high. One strategy for efficiently collecting data for parametrized problems that has arisen within the reduced basis literature is a greedy algorithm (Veroy et al. 2003; Cuong et al. 2005; Haasdonk 2013) based on *a posteriori* error estimates that are available for certain classes of problems (Grepl and Patera 2005; Veroy et al. 2003) or other error indicators (Paul-Dubois-Taine and Amsallem 2015). This procedure iteratively selects parameters at which to collect snapshot data by choosing the parameter value with the highest error estimate from a set of candidate parameters (usually a grid). For static problems, the new snapshot is simply orthogonalized and added to the existing basis. For dynamic problems, the leading POD modes of the new set of snapshots are added to basis (Grepl and Patera 2005; Haasdonk 2013). This snapshot-collection-and-basis-enrichment step is then repeated until the error estimates across all candidate parameters are below a user-specified tolerance.

In practice, a number of settings arise where a high number of basis vectors is required to achieve a state approximation with acceptably low error. The ROM literature often uses the language 'high Kolmogorov N-width' (see, e.g., Peherstorfer 2022 for a recent perspective) to describe the high maximum state error (the 'width') of an approximation with N basis vectors. Various strategies have been proposed to address this challenge in different settings. For example, when the Kolmogorov N-width is high due to large state variations across parameters, a number of works have proposed clustering strategies that develop ROMs for local regions of parameter space (Eftang et al. 2010; Haasdonk et al. 2011; Maday and Stamm 2013; Hess et al. 2019; Bonito et al. 2021; Peherstorfer et al. 2014; Amsallem et al. 2012; Amsallem and Haasdonk 2016). Transport-dominated problems also typically suffer from high Kolmogorov N-widths, leading to a number of ROM approaches based on coordinate transformations or time evolution of a small number of basis vectors (Rowley and Marsden 2000; Rowley et al. 2003; Peherstorfer 2020; Sesterhenn and Shahirpour 2019; Reiss et al. 2018a; Rim et al. 2018; Welper 2017a; Ohlberger and Rave 2013; Beyn and Thümmler 2004; Iollo and Lombardi 2014; Ehrlacher et al. 2020; Gerbeau and Lombardi 2014; Pagliantini 2021; Camacho and Huang 2025; Huang et al. 2022). *Registration*-based methods refer to another class of approaches developed for transport-dominated problems that apply (often nonlinear) transformations to the snapshot data to align features before using linear compression (Nair and Balajewicz 2019; Taddei 2020; Zucatti and Zahr 2024; Razavi and Yano 2025). Similarly, shifted proper orthogonal decomposition (Reiss et al. 2018b; Welper 2017b) and shifted Operator Inference (Issan and Kramer 2022) leverage shifts of the spatial variable to align shock features in the solutions before compression. More broadly, a number of approaches have been proposed that adapt the reduced basis and resulting ROM during the online phase based on newly obtained snapshot data (San and Borggaard 2015; Peherstorfer and Willcox 2015a, 2016b, 2015b; Kramer et al. 2017; Zimmermann et al. 2018; Weickum et al. 2009; Hesthaven et al. 2022a) or by splitting existing snapshots (Carlberg 2015).

Finally, several works have explored the inclusion of state derivatives within the approximation basis. Some of the earliest RB works actually define a local approximation basis using only a single state and its derivatives with respect to parameters (Noor and Peters 1981; Noor 1982, 1981; Noor and Peters 1980). The inclusion of parameter derivatives in the basis has been shown to be beneficial for optimization with respect to the parameters (Hay et al. 2009; Schmidt et al. 2013; Weickum et al. 2009). There are also works which use state derivative data to enrich the snapshot matrix before the POD basis is computed (García-Archilla et al. 2023; Koc et al. 2021a; Carlberg and Farhat 2011; Wen and Zahr 2023).

2.1.5 POD vs. RB methods

We conclude our introduction of POD and RB methods with a few remarks on how the methods are related but different. The two methodological families have different historical origins: POD initially grew out of work in modal analysis of fluid flows (Lumley 1967; Berkooz et al. 1993), whereas RB methods originated in structural analysis (Fox and Miura 1971; Almroth et al. 1978; Noor and Peters 1980). Much of the mathematical reduced basis literature has considered highly structured parametrized problems for which error bounds may be derived (Fink and Rheinboldt 1983; Nagy 1979;

Porsching and Lee 1987; Porsching 1985), analogous to the error analysis of finite element methods (Gunzburger 2012). In recent decades, the term 'reduced basis method' therefore has largely been used to refer to methods which use a greedy procedure based on these error bounds to build the reduced basis, and exploit affine parameter dependence for efficient online computation (Rozza et al. 2008; Hesthaven et al. 2022a). The term 'POD,' in contrast, has been used to describe any approach in which the approximation basis is defined to be the principal components of the available snapshots—indeed, RB methods for time-dependent systems tend to refer to the enrichment of the basis with POD modes at greedily chosen parameter values as the 'POD-Greedy' procedure (Grepl and Patera 2005; Haasdonk and Ohlberger 2008; Haasdonk 2013). Recent works in POD and RB methods now often combine elements historically associated with both research communities.

2.2 Nonlinear model reduction and hyperreduction

We now discuss some of the specific challenges in developing computationally efficient ROMs for nonlinear models. Section 2.2.1 describes the computational bottleneck that arises in model reduction of general nonlinear models. Section 2.2.2 then describes how low-order polynomial models may be efficiently reduced, and how variable transformations may be used to transform general nonlinearities into polynomial models. Section 2.2.3 then describes *hyperreduction* strategies that lower the computational cost of evaluating general nonlinear ROMs by introducing an additional layer of approximation.

2.2.1 The computational bottleneck of nonlinear model eduction

As we discussed in Sect. 2.1.3, definition of a POD/RB ROM via Galerkin projection leads to a *computational bottleneck* for nonlinear systems, because evaluation of the projected governing equations requires evaluation of the high-dimensional nonlinear operator $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$. Similar circumstances occur in the related class of least-squares Petrov–Galerkin (LSPG) methods, which instead of defining the ROM through an explicit algebraic expression, see Sect. 2.1.3, define the ROM *implicitly* through the minimization of a discrete-time residual (Carlberg et al. 2011, 2017; Parish et al. 2020). Solution of this minimization requires evaluation of the residual of the high-dimensional governing equation. Thus, projection of a nonlinear model onto a low-dimensional reduced basis is generally insufficient to guarantee computational efficiency of the resulting ROM.

2.2.2 Polynomial nonlinearities and efficiency through variable transformations

Consider the following nonlinear dynamical system with *polynomial* nonlinearities in the system state:

$$\dot{\mathbf{x}} = \mathbf{A}^{(1)}\mathbf{x} + \mathbf{A}^{(2)}(\mathbf{x} \otimes \mathbf{x}) + \cdots + \mathbf{A}^{(p)}(\underbrace{\mathbf{x} \otimes \cdots \otimes \mathbf{x}}_{p \text{ times}}), \tag{8}$$

where $\otimes$ denotes the Kronecker product, and $\mathbf{A}^{(m)} \in \mathbb{R}^{n \times n^m}$ denotes a matricized tensor operator of order $m+1$ containing coefficients of the order-m polynomial terms of the state $\mathbf{x}$. Galerkin (and Petrov–Galerkin) projection preserves this polynomial structure (Benner and Breiten 2015; Kramer and Willcox 2019): substituting the state approximation $\mathbf{x} \approx \mathbf{U}_r\hat{\mathbf{x}}$ into (8) and enforcing the Galerkin orthogonality condition yields

$$\dot{\hat{\mathbf{x}}} = \hat{\mathbf{A}}^{(1)}\hat{\mathbf{x}} + \hat{\mathbf{A}}^{(2)}(\hat{\mathbf{x}} \otimes \hat{\mathbf{x}}) + \cdots + \hat{\mathbf{A}}^{(p)}(\underbrace{\hat{\mathbf{x}} \otimes \cdots \otimes \hat{\mathbf{x}}}_{p \text{ times}}), \tag{9}$$

where $\hat{\mathbf{A}}^{(m)} \in \mathbb{R}^{r \times r^m}$ denotes the matricized tensor *reduced* polynomial operator of order $m+1$, which is given by (via properties of the Kronecker product (Van Loan 2000))

$$\hat{\mathbf{A}}^{(m)} = \mathbf{U}_r^\top \mathbf{A}(\underbrace{\mathbf{U}_r \otimes \cdots \otimes \mathbf{U}_r}_{m \text{ times}}).$$

Computationally, this projection can be implemented with efficient memory usage following (Benner and Breiten 2015; Kolda and Bader 2009). Thus, we see that projection-based model reduction of polynomial nonlinear terms preserves the polynomial structure of the operators, and the polynomial reduced operators scale only with the reduced dimension r and not the original high dimension n. However, the overall cost of evaluating ROM scales with r^p, which grows quickly with the maximum polynomial order p. For this reason, polynomial ROMs that have been successful in the literature are generally limited to low-order polynomial terms.

When a nonlinear model contains non-polynomial nonlinear terms, projection of these non-polynomial terms leads to the computational bottleneck described in Sect. 2.2.1. One approach to approximate non-polynomial nonlinearities is to perform a local Taylor expansion of the model, which may be accurate only within a small radius of the expansion point. An alternative approach uses variable transformations, sometimes including the introduction of auxiliary variables, to *exactly* transform a nonlinear system to polynomial form: In particular, a large class of nonlinearities may be converted to quadratic form via the introduction of auxiliary variables (Gu 2011; Bychkov et al. 2024). Such transformations have been used for efficient computation of intrusive projection-based ROMs in (Benner and Breiten 2015; Kramer and

Willcox 2019; Kramer et al. 2022; Ritschel et al. 2020). However, one challenge in exploiting such variable transformations for intrusive model reduction is that computing the reduced operators $\hat{\mathbf{A}}^{(m)}$ requires access to the transformed polynomial operators $\mathbf{A}^{(m)}$, which are often not available because the original high-dimensional model is written in the original variables, not the transformed variables. For this reason, such variable transformations are often combined with non-intrusive model reduction strategies, as introduced in Sect. 2.3.

2.2.3 Hyperreduction

The term 'hyperreduction' in model reduction refers to strategies that introduce an additional layer of approximation *beyond* the initial projection of the governing equations in order to circumvent the computational bottleneck that arises when non-polynomial nonlinearities are projected onto the reduced basis. The discrete empirical interpolation method (DEIM) (Chaturantabut and Sorensen 2010) builds on Barrault et al. (2004) by exploiting *local* structure in the nonlinear function $\mathbf{f}$, meaning that a low number of entries of $\mathbf{f}(\mathbf{x}) \in \mathbb{R}^n$ may be evaluated *without* requiring construction of the full high-dimensional vector $\mathbf{x} \in \mathbb{R}^n$. This local structure arises, for example, in pointwise nonlinear terms, or in nonlinear terms that depend on derivatives which are approximated using stencils that require access to only a few entries of the vector $\mathbf{x}$. DEIM then computes an *interpolatory* projection operator which approximates $\mathbf{f}(\mathbf{x})$ from a few of its entries—the interpolation points. While the work (Chaturantabut and Sorensen 2010) originally proposed a greedy algorithm for selecting interpolation points, later work (Drmac and Gugercin 2016) showed that selecting points using the permutation operator obtained from a QR decomposition with column pivoting can outperform the greedy approach. Further developments of DEIM include the extension to oversampling the vector $\mathbf{f}(\mathbf{x})$, which yields approximation of the nonlinear term via regression rather than interpolation and can be more robust to noise (Peherstorfer et al. 2020); as well as local (Peherstorfer et al. 2014) and adaptive (Peherstorfer and Willcox 2015b) approaches. Related approaches including missing point estimation (Astrid et al. 2008), best points interpolation (Nguyen et al. 2008; Nguyen and Peraire 2008), and first-order EIM (Nguyen and Peraire 2023), which incorporates parameter derivative information into the interpolatory approximation. While the previous methods directly approximate the nonlinear term of the model, the Gauss–Newton approximate tensor (GNAT) approach considers the case where a Gauss–Newton method is used to solve the nonlinear system (Carlberg et al. 2011) and applies a hyperreduction strategy to the residual and action of the Jacobian inside the Gauss–Newton method.

The strategies discussed thus far can be classified as 'approximate-then-project' strategies because the hyperreduction strategy is applied to the discretized model. 'Project-then-approximate' strategies are also possible, i.e., developing the projection for the continuous formulation and then discretizing. For example, the energy-conserving mesh sampling and weighting (ECSW) approach (Farhat et al. 2014) is a project-then-approximate approach that directly approximates the reduced residuals and Jacobians within a nonlinear solve (additional structure-preserving hyperreduction approaches are discussed in Sect. 4.3). The empirical cubature approaches for hyperreduction of nonlinear residuals (Hernandez et al. 2017; Yano 2019) propose an optimization problem to reduce the error in integrating reduced internal forces.

2.3 Non-intrusive model reduction

Our discussion of state approximation model reduction methods has thus far focused on *intrusive* model reduction methods, which require access to the operators of the full-order model (FOM), e.g., $\mathbf{A}(\boldsymbol{\mu})$ or $\mathbf{f}(\cdot;\boldsymbol{\mu})$ in (1), in order to compute their reduced approximations. In contrast, *non-intrusive* state approximation methods for model reduction approximate reduced operators from system state data, without requiring access to the operators of the FOM. This makes non-intrusive approaches an attractive alternative to intrusive approaches in settings where operators of the FOM are not available, for example, when only a state data set is available, or when the FOM is a black box or otherwise inaccessible. Non-intrusive model reduction approaches can be viewed as machine learning methods, and indeed popular machine learning methods including Gaussian process regression and neural networks are often combined with elements of state approximation in a reduced dimension to propose new methods.

Note that in state approximation model reduction methods, the POD/RB basis is *already* obtained from available state data. What distinguishes non-intrusive methods from intrusive methods is that non-intrusive methods additionally fit the reduced operators to data. A natural way to do this for linear and polynomial models is via a linear regression formulation, where the reduced matrix operators solve a linear least-squares problem, as done in the Operator Inference method (Peherstorfer and Willcox 2016a). There have been many developments of Operator Inference beyond the basic formulation, including its application to non-polynomial models via quadratizing variable transformations (Qian et al. 2020), the use of regularization strategies (McQuarrie et al. 2021), structure-preserving versions (Harsh et al. 2025; Geng et al. 2025a, 2024; Sharma and Kramer 2024; Sharma et al. 2024b), and extensions to work with oblique projections (Padovan et al. 2024). Moreover, there are several

extensions such as working on distributed computing platforms (Farcaş et al. 2025) and domain decomposition (Farcaş et al. 2024) that have demonstrated scalability to over 75 million degrees of freedom. We refer to (Kramer et al. 2024b) for a recent review.

Operator Inference is typically formulated for continuous-time dynamical systems. For discrete-time dynamical systems, model reduction methods based on the dynamic mode decomposition (DMD) (Koopman 1931; Mezić 2005; Rowley et al. 2009; Schmid 2010, 2022; Kutz et al. 2016; Lu and Tartakovsky 2021, 2020) fit a linear operator to the leading POD modes of discrete time series data. DMD-based methods can be extended to nonlinear systems through Koopman operator theory (Korda and Mezić 2018; Mezić 2005, 2013; Schmid 2010; Williams et al. 2015), which states that any finite-dimensional nonlinear dynamical system is equivalent to an infinite-dimensional linear dynamical system that describes the evolution of *observables* (functions of) the system state.

In contrast to Operator Inference and DMD methods, which both learn structured matrix operators from data, it is also possible to learn more general nonlinear functions which describe the dynamics of the reduced state. Many works have explored the use of neural networks to approximate the dynamics in the reduced space (Swischuk et al. 2019; Bhattacharya et al. 2021; Yu and Hesthaven 2019; Mainini and Willcox 2017; Wang et al. 2019; Hijazi et al. 2023). The use of Gaussian process regression models, also referred to as Kriging models, is also popular (Ortali et al. 2022; Yang and Xiao 2020; Guo and Hesthaven 2018). Recently, there has been growing interest in using machine learning tools to learn *nonlinear* reduced dimension encoders of the high-dimensional state (cf. again to Fig. 1), for example using quadratic manifolds (Barnett and Farhat 2022; Geelen et al. 2023, 2024; Schwerdtner and Peherstorfer 2024; Sharma et al. 2023), or neural network autoencoders (Lee and Carlberg 2020). One body of work that frames the non-intrusive model learning problem as a combination of both learning a reduced representation of the state combined with the identification of dynamics within this latent space is called latent space dynamics identification (LaSDI) (Fries et al. 2022; Bonneville et al. 2024). Other approaches include the use of radial basis functions (Audouze et al. 2013), nearest neighbors (Swischuk et al. 2019), or sparse regression (Loiseau and Brunton 2018) to fit the model dynamics in the low-dimensional latent space.

3 System-theoretic model reduction

The field of dynamical systems and control theory demonstrated a need for model reduction in the 1970s, as high-dimensional semi-discretizations of distributed parameter systems (here: PDEs) became modeling practice, yet the prevalent contemporary control methods would not scale to their high dimensions. Model reduction through the lens of systems theory became an early active area of research, a trend which has continued as controlled systems become ever more complex.

Methods for linear time-invariant (LTI) systems are relatively mature at present, and research focuses on nonlinear and/or structured systems: control-affine control systems, quadratic and polynomial drift systems; structured systems of (port-) Hamiltonian, Lagrangian, positive-real, and energy-conserving natures. Moreover, current research continues to span both the frequency/Laplace-domain as well as time domain.

Throughout this section, we outline the key ideas behind each method and intuition for how to construct subspaces based on LTI systems. We will then provide a detailed discussion of these methods applied to nonlinear systems model reduction, and discuss very recent trends in this area. Section 3.1 starts by defining the model setting, followed by Sect. 3.2 presenting common approaches in interpolatory model reduction. In Sect. 3.3 we illustrate and survey the concept of balanced truncation model reduction, and Sect. 3.4 focuses on practical system-theoretic data-driven model reduction methods in the frequency domain, and Sect. 3.5 in the time domain. We close the system-theoretic section with a discussion of moment-matching methods in Section 3.2.

3.1 Setting

We consider a parameter-dependent and control-affine state-space model

$$\mathbf{E}(\boldsymbol{\mu})\dot{\mathbf{x}}(t;\boldsymbol{\mu}) = \mathbf{f}(\mathbf{x},\boldsymbol{\mu}) + \mathbf{g}(\mathbf{x};\boldsymbol{\mu})\mathbf{u}(t) \tag{10}$$

$$\mathbf{y}(t;\boldsymbol{\mu}) = \mathbf{h}(\mathbf{x};\boldsymbol{\mu}) + \mathbf{D}(\boldsymbol{\mu})\mathbf{u}(t) \tag{11}$$

where $\mathbf{x}(t;\boldsymbol{\mu}) \in \mathbb{R}^n$ is the high-dimensional system state, $t \geq 0$ denotes time, the initial condition is $\mathbf{x}(0) = \mathbf{x}_0$, $\boldsymbol{\mu} \in \mathcal{D}$ is a parameter vector with domain $\mathcal{D}$, and $\mathbf{f} : \mathbb{R}^n \times \mathcal{D} \mapsto \mathbb{R}^n$ is the nonlinear drift, $\mathbf{g} : \mathbb{R}^n \times \mathcal{D} \mapsto \mathbb{R}^{n\times m}$ is the input-to-state operator, $\mathbf{u}(t) \in \mathbb{R}^m$ is the vector of time-dependent inputs or controls, $\mathbf{y}(t) \in \mathbb{R}^p$ are the outputs with $\mathbf{h} : \mathbb{R}^n \times \mathcal{D} \mapsto \mathbb{R}^p$ is the output mapping, $\mathbf{E}(\boldsymbol{\mu}) \in \mathbb{R}^{n\times n}$ is the mass matrix, and $\mathbf{D}(\boldsymbol{\mu}) \in \mathbb{R}^{p\times m}$ is the feedthrough matrix operator. In the context of this section, we call equations (10)–(11) the full-order model (FOM), or high-fidelity model.

As our brief exposition introduces the key ideas in the context of LTI systems, which are often obtained by linearizing equations (10)–(11), we introduce those here:

$$\mathbf{E}(\boldsymbol{\mu})\dot{\mathbf{x}}(t;\boldsymbol{\mu}) = \mathbf{A}(\boldsymbol{\mu})\mathbf{x}(t;\boldsymbol{\mu}) + \mathbf{B}(\boldsymbol{\mu})\mathbf{u}(t) \tag{12}$$

$$\mathbf{y}(t;\boldsymbol{\mu}) = \mathbf{C}(\boldsymbol{\mu})\mathbf{x}(t;\boldsymbol{\mu}) + \mathbf{D}(\boldsymbol{\mu})\mathbf{u}(t), \tag{13}$$

where $\mathbf{A}(\boldsymbol{\mu}) \in \mathbb{R}^{n\times n}$ is the linear system matrix operator, $\mathbf{B}(\boldsymbol{\mu}) \in \mathbb{R}^{n\times m}$ is the input operator matrix, $\mathbf{C}(\boldsymbol{\mu}) \in \mathbb{R}^{p\times n}$ is the output operator matrix. These matrices are obtained via first-order Taylor series of the nonlinear terms around an expansion point $\bar{\mathbf{x}}$, i.e., $[\mathbf{A}(\boldsymbol{\mu})]_{i,j} = \frac{\partial \mathbf{f}_i}{\partial x_j}\Big|_{\mathbf{x}=\bar{\mathbf{x}}}$, $\mathbf{B}(\boldsymbol{\mu}) = \mathbf{g}(\bar{\mathbf{x}},\boldsymbol{\mu})$, $[\mathbf{C}(\boldsymbol{\mu})]_{i,j} = \frac{\partial \mathbf{h}_i}{\partial x_j}\Big|_{\mathbf{x}=\bar{\mathbf{x}}}$.

3.2 Interpolatory model reduction

Interpolatory model reduction considers the input–output map of a dynamical system, given by the transfer function, and approximates it with a lower-degree ROM transfer function. To illustrate the core idea, consider the Laplace transform of the LTI system (12)–(13) with zero initial conditions and fixed $\boldsymbol{\mu}$, which yields

$$\begin{aligned}&\mathbf{Y}(s;\boldsymbol{\mu}) = \mathbf{H}(s;\boldsymbol{\mu})\mathbf{U}(s) \quad \text{with}\\ &\mathbf{H}(s;\boldsymbol{\mu}) = \mathbf{C}(\boldsymbol{\mu})(s\mathbf{E}(\boldsymbol{\mu}) - \mathbf{A}(\boldsymbol{\mu}))^{-1}\mathbf{B}(\boldsymbol{\mu})\end{aligned} \tag{14}$$

for the Laplace variable $s \in \mathbb{C}$ and $\mathbf{U}(s) = \mathcal{L}(\mathbf{u}(t))$ the Laplace-transformed input. A ROM with similar structured transfer function, but of lower order (e.g., number of poles) is

$$\begin{aligned}&\hat{\mathbf{Y}}(s;\boldsymbol{\mu}) = \hat{\mathbf{H}}(s;\boldsymbol{\mu})\mathbf{U}(s) \quad \text{with}\\ &\hat{\mathbf{H}}(s;\boldsymbol{\mu}) = \hat{\mathbf{C}}(\boldsymbol{\mu})(s\hat{\mathbf{E}}(\boldsymbol{\mu}) - \hat{\mathbf{A}}(\boldsymbol{\mu}))^{-1}\hat{\mathbf{B}}(\boldsymbol{\mu}).\end{aligned} \tag{15}$$

where now $\hat{\mathbf{C}}(\boldsymbol{\mu}) \in \mathbb{R}^{p\times r}$, $\hat{\mathbf{A}}(\boldsymbol{\mu}) \in \mathbb{R}^{r\times r}$, $\hat{\mathbf{B}}(\boldsymbol{\mu}) \in \mathbb{R}^{r\times m}$, and r is the reduced order. Interpolatory ROMs, as their name suggests, interpolate the FOM at a select number of points s_i, so $\mathbf{H}(s_i;\boldsymbol{\mu}) = \hat{\mathbf{H}}(s_i;\boldsymbol{\mu})$ for $i = 1,\ldots,2r$ in the simplest single-input single-output case. Interpolation of multiple-input multiple-output systems is mostly done along tangential directions. Further interpolation conditions for the derivatives of the transfer function (the moments) can also be enforced, as we will see below.

Optimal $\mathcal{H}_2$ and $\mathcal{H}_\infty$ interpolatory ROMs

To design the ROM transfer function, the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ frameworks consider the error of the output from the FOM and the ROM in the time domain. Its upper bound includes the frequency-domain error of the two transfer functions as

$$\begin{aligned}&\|\mathbf{y}(t;\boldsymbol{\mu}) - \hat{\mathbf{y}}(t;\boldsymbol{\mu})\|_{L_2((0,T])}\\ &\quad\le \|\mathbf{H}(s;\boldsymbol{\mu}) - \hat{\mathbf{H}}(s;\boldsymbol{\mu})\|_{\mathcal{H}_\infty}\|\mathbf{u}(t)\|_{L_2((0,T])},\end{aligned} \tag{16}$$

$$\begin{aligned}&\|\mathbf{y}(t;\boldsymbol{\mu}) - \hat{\mathbf{y}}(t;\boldsymbol{\mu})\|_{L_\infty((0,T])}\\ &\quad\le \|\mathbf{H}(s;\boldsymbol{\mu}) - \hat{\mathbf{H}}(s;\boldsymbol{\mu})\|_{\mathcal{H}_2}\|\mathbf{u}(t)\|_{L_\infty((0,T])}\end{aligned} \tag{17}$$

where the norms in the Hardy spaces $\mathcal{H}_2$ and $\mathcal{H}_\infty$ are defined as

$$\|\mathbf{H}(s;\boldsymbol{\mu}) - \hat{\mathbf{H}}(s;\boldsymbol{\mu})\|_{\mathcal{H}_\infty} := \sup_{\omega\in\mathbb{R}} \|\mathbf{H}(j\omega;\boldsymbol{\mu}) - \hat{\mathbf{H}}(j\omega;\boldsymbol{\mu})\|_2, \tag{18}$$

$$\|\mathbf{H}(s;\boldsymbol{\mu}) - \hat{\mathbf{H}}(s;\boldsymbol{\mu})\|_{\mathcal{H}_2} := \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} \|\mathbf{H}(j\omega;\boldsymbol{\mu}) - \hat{\mathbf{H}}(j\omega;\boldsymbol{\mu})\|_F \,\mathrm{d}\omega\right)^{1/2}, \tag{19}$$

which can be efficiently computed, see, e.g., (Krajewski et al. 1995) and also (Antoulas 2005) for a detailed derivation. These output error bounds are at the heart of the two model reduction procedures that minimize the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms. While early work by (Meier and Luenberger 1967) derived the interpolation conditions for general transfer functions, work by (Feldmann and Freund 1995; Grimme 1997; Gallivan et al. 2006) showed that in the context of ROMs, Hermite interpolation (e.g., also interpolating the first moment) is possible without explicit computation of the quantities to be matched. It took several decades of work until (Gugercin et al. 2008) proposed the iterative rational Krylov algorithm (IRKA), which, if converged, recovers the optimality conditions. It is also often practical to introduce certain weighted norms in the frequency space to ensure ROMs are more accurate over desired ranges, as done in (Anić et al. 2013; Breiten et al. 2015; Vuillemin et al. 2013). We refer the interested reader to the recent book (Antoulas et al. 2020) on interpolatory model reduction for a detailed historical account, an accurate snapshot of the state of the art and a detailed discussion of the parametric case.

Moment matching

Matching the moments of an LTI system is another form of interpolatory ROM construction. Considering LTI systems, the kth moment of a system (12)–(13) at $s = s_i$ is defined as

$$\mathbf{m}_k(s_i) := \frac{(-1)^k}{k!}\left[\frac{\mathrm{d}^k}{\mathrm{d}s^k}\mathbf{H}(s)\right]_{s=s_i} \tag{20}$$

where s_i is not in the spectrum of $\mathbf{A}$. The moments of an LTI system can be characterized through the solution of a Sylvester equation, see (Gallivan et al. 2004, 2006), which led to the observation that the moments have a one-to-one relationship with the steady-state output response (provided it exists) of the system when excited by a certain signal generator (Scarciotti and Astolfi 2017; Astolfi et al. 2020). An early survey of moment-matching methods for coupled systems is given in (Reis et al. 2008), and (Scarciotti and Astolfi 2017; Astolfi et al. 2020) provide a more recent survey on moment matching for nonlinear systems. Matching moments of linear systems can be achieved in both the time and frequency domain, where Krylov subspaces emerge again as the right choice of projection spaces. There are extensions

to linear parameter varying systems (Başţuäÿ et al. 2015) and a simple and robust algorithm for matching an arbitrary number of moments for parametric LTI systems (Benner and Feng 2014). Instead of forcing equality in the moment interpolation, (Padoan 2023) proposes that the interpolation conditions imposed by moment matching may only have to be satisfied in a least-squares sense, leading to a less restrictive problem and allowing for error analysis.

Extensions to the nonlinear case

Interpolatory model reduction for systems beyond LTI is an active area of research. The previous moment-matching concept has been extended via signal generators to the nonlinear case in (Scarciotti and Astolfi 2017; Astolfi et al. 2020). The works (Benner and Breiten 2012; Flagg and Gugercin 2015) proposed $\mathcal{H}_2$ model reduction for bilinear systems, (Benner et al. 2018) extended this to quadratic-bilinear systems, and finally (Benner and Goyal 2021) covered the polynomially nonlinear case. We additionally discuss structure-preserving methods for linear and nonlinear systems in Sect. 4.3 and data-driven model reduction based on interpolation principles, both for linear and nonlinear systems, in Sect. 3.4.

3.3 Balanced truncation model reduction

Balanced truncation model reduction, as pioneered in (Moore 1981) and (Mullis and Roberts 1976) for LTI systems, uses the controllability and observability energies of a system to determine the states that are most important for controller and observer design. If a state requires a large amount of input energy to be reached and also has minimal effect on the output, then the ROM would neglect that state without a significant impact on the input–output behavior of the system. We restrict our discussion to the non-parametric case.

The controllability and observability energy functions are defined as

$$\mathcal{E}_c(\mathbf{x}_0) := \min_{\substack{\mathbf{u}\in L_2(-\infty,0]\\ \mathbf{x}(-\infty)=\mathbf{0}\\ \mathbf{x}(0)=\mathbf{x}_0}} \frac{1}{2}\int_{-\infty}^{0} \|\mathbf{u}(t)\|^2 \mathrm{d}t,$$

$$\mathcal{E}_o(\mathbf{x}_0) := \frac{1}{2}\int_{0}^{\infty} \|\mathbf{y}(t)\|^2 \mathrm{d}t, \quad \mathbf{u}(t) \equiv \mathbf{0}.$$

The controllability energy quantifies for every state $\mathbf{x}_0$ the minimal control energy required to reach that state from $\mathbf{x}(-\infty) = \mathbf{0}$. The observability energy function quantifies the output energy produced by a state $\mathbf{x}_0$.

For an asymptotically stable, controllable, and observable LTI system, these energies are given by

$$\mathcal{E}_c(\mathbf{x}_0) = \mathbf{x}_0^\top \mathbf{P}^{-1}\mathbf{x}_0, \qquad \mathcal{E}_o(\mathbf{x}_0) = \mathbf{x}_0^\top \mathbf{Q}\mathbf{x}_0 \tag{21}$$

where $\mathbf{P} := \int_0^\infty e^{\mathbf{A}t}\mathbf{B}\mathbf{B}^\top e^{\mathbf{A}^\top t}\mathrm{d}t$ and $\mathbf{Q} := \int_0^\infty e^{\mathbf{A}^\top t}\mathbf{C}^\top\mathbf{C}e^{\mathbf{A}t}\mathrm{d}t$ are the controllability and observability Gramian, respectively. The Gramians are symmetric positive definite ($\mathbf{P} = \mathbf{P}^\top > \mathbf{0}$ and $\mathbf{Q} = \mathbf{Q}^\top > \mathbf{0}$) and satisfy the continuous-time Lyapunov equations:

$$\mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^\top + \mathbf{B}\mathbf{B}^\top = \mathbf{0}, \qquad \mathbf{A}^\top\mathbf{Q} + \mathbf{Q}\mathbf{A} + \mathbf{C}^\top\mathbf{C} = \mathbf{0}. \tag{22}$$

The *balancing* task is, from a linear algebra point of view, concerned with finding a transformed coordinate system in which the transformed Gramians are equal and diagonal. That is, for some invertible transformation $\mathbf{T}$, let $\tilde{\mathbf{x}}_0 = \mathbf{T}^{-1}\mathbf{x}_0$. We seek $\mathbf{T}$ satisfying

$$\mathcal{E}_c(\tilde{\mathbf{x}}_0) = \tilde{\mathbf{x}}_0^\top \boldsymbol{\Sigma}^{-1}\tilde{\mathbf{x}}_0, \qquad \mathcal{E}_o(\tilde{\mathbf{x}}_0) = \tilde{\mathbf{x}}_0^\top \boldsymbol{\Sigma}\tilde{\mathbf{x}}_0 \tag{23}$$

for some diagonal positive-definite $\boldsymbol{\Sigma}$. This coordinate system is called a balanced coordinate system. In this coordinate system, the leading coordinates simultaneously exhibit the lowest controllability and highest observability energies, enabling us to then *truncate* the system to model only the leading coordinates. In practice, instead of first computing a full transformation matrix $\mathbf{T}$ and then truncating, which can lead to computational problems related to ill-conditioning (Opmeer 2015; Baker et al. 2015), the following simultaneous balance-and-reduce strategy is employed:

1. Solve the Lyapunov equations (22) for $\mathbf{P} \approx \mathbf{R}\mathbf{R}^\top$, $\mathbf{Q} \approx \mathbf{L}\mathbf{L}^\top$ either via a low-rank solver, or a direct solver with subsequent Cholesky factorization.
2. Compute the singular value decomposition $\mathbf{L}^\top\mathbf{R} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top$ and set $\mathbf{U}_r = \mathbf{U}(:, 1:r)$, $\boldsymbol{\Sigma}_r = \boldsymbol{\Sigma}(1:r, 1:r)$, $\mathbf{V}_r = \mathbf{V}(:, 1:r)$.
3. Set $\mathbf{T}_r := \mathbf{R}\mathbf{V}_r\boldsymbol{\Sigma}_r^{-\frac{1}{2}}$ and left inverse $\mathbf{T}_r^+ = \boldsymbol{\Sigma}_r^{-\frac{1}{2}}\mathbf{U}_r^\top\mathbf{L}^\top$.
4. The balanced model is then $\hat{\mathbf{A}} = \mathbf{T}_r^+\mathbf{A}\mathbf{T}_r$, $\hat{\mathbf{B}} = \mathbf{T}_r^+\mathbf{B}$, $\hat{\mathbf{C}} = \mathbf{C}\mathbf{T}_r$.

One of the appealing factors of balanced truncation is its a priori error bound. Consider an asymptotically stable and balanced LTI system with controllability and observability Gramian $\mathbf{P} = \mathbf{Q} = \mathrm{diag}(\sigma_1\mathbf{I}_{s_1}, \sigma_2\mathbf{I}_{s_2}, \ldots, \sigma_k\mathbf{I}_{s_k})$($\sigma$ could be repeated), where $\sigma_1 > \sigma_2 > \ldots > \sigma_k \geqslant \mathbf{0}$. Let $[\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{D}}]$ be the ROM obtained via projection with $r = s_1 + s_2 + \ldots + s_l$ for some $l \leq k$. Then (Enns 1984; Glover 1984):

$$||\mathbf{H} - \hat{\mathbf{H}}||_{\mathcal{H}_\infty} \leqslant \sum_{j=l+1}^{k} 2\sigma_j. \tag{24}$$

Inserting this into the output bounds in (16)–(17), we see that the output of a BT ROM can be bounded via the computed Hankel singular values and the energy of the input function.

Balancing beyond the standard Gramians

While the above derivation assumes stable LTI systems with no particular structure, extensions to closed-loop LTI sys-

tems led to the LQG balancing (Verriest 1981; Jonckheere and Silverman 1983) and $\mathcal{H}_\infty$ balancing concepts (Mustafa and Glover 1991). If there is additional structure in the system model, specific techniques also exist, such as stochastic balancing (Desai and Pal 1984; Green 1988), bounded real balancing (Opdenacker and Jonckheere 1988), positive-real balancing (Desai and Pal 1984; Ober 1991), and frequency-weighted balancing (Enns 1984); see also the surveys (Gugercin and Antoulas 2004; Benner and Breiten 2017). Due to synergistic development of computational methods for solving large-scale Lyapunov or Riccati-type algebraic matrix equations via low-rank solvers (Benner and Saak 2013; Simoncini 2016; Saak et al. 2025) and doubling methods (Li et al. 2013), it is possible to apply LTI balancing for systems with millions of states. A snapshot-based balanced truncation method with frequency-domain snapshots to approximate the Gramians (only requiring linear systems solves) was first proposed in (Willcox and Peraire 2002), and a time-domain snapshot-based computation in (Rowley 2005).

Balanced truncation for nonlinear systems

The theoretical foundations for balanced truncation of nonlinear open-loop systems were laid in (Scherpen 1993) for stable systems, showing that the respective energy functions can be computed as solutions to Hamilton–Jacobi PDEs. Theoretical extensions to unstable systems by considering the closed-loop setting, such as Hamilton–Jacobi–Bellman (HJB) balancing (Scherpen and Van der Schaft 1994) and $\mathcal{H}_\infty$ balancing (Scherpen 1996), have also been proposed. Until recently, this framework was merely applicable to systems with a handful of degrees of freedom, yet advances in numerical methods for solving Hamilton–Jacobi–Bellman equations (Borggaard and Zietsman 2020; Kramer et al. 2024a; Corbin and Kramer 2025) and the nonlinear balancing transformations (Corbin et al. 2025) are promising towards truly nonlinearly balanced ROMs (Kramer et al. 2023). It is also possible to balance slightly different systems, such as the incremental system (Besselink et al. 2014) (for shorter periods of time), for which more rigorous theory can be derived. In addition, several extensions that consider structure preservation in combination with balancing, such as dissipativity-preserving (Ionescu et al. 2010) and positive-real balancing (Ionescu et al. 2009) for nonlinear systems have been proposed. A framework for balancing LTI systems with quadratic (energy-like) outputs has been proposed in Benner et al. (2021).

There are also data-driven versions of nonlinear balancing, starting with the idea to use empirical Gramians in (Lall et al. 2002) to (Otto et al. 2023), where the authors use information of the state as well as the adjoint of the system, and balance the Gramian matrices that are approximated from snapshots of both of these simulations. Moreover, the TROOP method (Otto et al. 2022) optimizes over subspaces to find the best linear subspaces to achieve balanced nonlinear models.

3.4 Data-driven system-theoretic model reduction from frequency-response measurements

While $\mathcal{H}_2$ and $\mathcal{H}_\infty$ ROMs and balanced ROMs are mainly constructed using a given state-space model and desired system-theoretic objectives, there are many situations in engineering where only input/output data—in the time or frequency domain—can be generated, be it from simulations of closed-source computer code or physical experiments. The goal then is to learn a ROM directly from that data, which has several connections to subspace-based system identification (Verhaegen 2013). We discuss a few popular and representative system-theoretic approaches for data-driven model reduction from frequency-domain data.

Loewner

The Loewner framework (Mayo and Antoulas 2007), here illustrated for multi-input multi-output non-parametric systems, constructs a rational interpolant in state-space form directly from data. It starts by assuming that we can obtain right (column) data $(\lambda_i, \mathbf{r}_i, \mathbf{w}_i) \in \mathbb{C} \times \mathbb{C}^m \times \mathbb{C}^p$, $i = 1, \ldots, k$ and left (row) data $(\mu_j, \mathbf{l}_j^*, \mathbf{v}_j^*) \in \mathbb{C} \times \mathbb{C}^p \times \mathbb{C}^m$, $j = 1, \ldots, q$, and we assume $\{\lambda_1, \ldots, \lambda_k\} \cap \{\mu_1, \ldots, \mu_q\} = \emptyset$. We seek a transfer function $\mathbf{H}(s)$ with values in $\mathbb{C}^{p\times m}$ such that

$$\mathbf{H}(\lambda_i)\mathbf{r}_i = \mathbf{w}_i, \quad i = 1, \ldots, k, \qquad \mathbf{l}_j^*\mathbf{H}(\mu_j) = \mathbf{v}_j^*, \quad j = 1, \ldots, q. \tag{25}$$

The Loewner framework uses the measurements to construct an LTI state-space model whose transfer function interpolates the data. To formalize this, let $k = q$ and the Loewner pencil $s\mathbb{L} - \mathbb{L}_\sigma$ be regular and assume that $\lambda_1, \ldots, \lambda_k$ and $\mu_1, \ldots, \mu_k$ are not eigenvalues of the pencil $(s\mathbb{L} - \mathbb{L}_\sigma)$. Then

$$\widetilde{\mathbf{E}} = -\mathbb{L}, \quad \widetilde{\mathbf{A}} = -\mathbb{L}_\sigma, \quad \widetilde{\mathbf{B}} = [\mathbf{v}_1^* \ \mathbf{v}_2^* \ \ldots \ \mathbf{v}_q^*],$$
$$\widetilde{\mathbf{C}} = [\mathbf{w}_1 \ \mathbf{w}_2 \ \ldots \ \mathbf{w}_k]$$

is an interpolating descriptor system realization, i.e., the function $\mathbf{H}(s) := \widetilde{\mathbf{C}}(s\widetilde{\mathbf{E}} - \widetilde{\mathbf{A}})^{-1}\widetilde{\mathbf{B}}$ interpolates the data at the given frequencies.

In practice, we often have enough data such that the Loewner pencil may be singular. To address this issue, a rank reduction step can be performed to obtain a reduced-order transfer function. Moreover, similar to the $\mathcal{H}_2$-optimal approximations, Hermite interpolation conditions can also be enforced, see, e.g., (Antoulas et al. 2020, Sec. 4.4).

We highlight some extensions to the Loewner framework for special classes of systems, such as those with parametric dependence (Ionita and Antoulas 2014), bilinear

systems (Antoulas et al. 2016), and quadratic-bilinear systems (Gosea and Antoulas 2018). The authors in Schulze et al. (2018) extend Loewner to more general forms of structured systems, including those with time delays. Moreover, the work (Beattie and Gugercin 2012) employs a Loewner-matrix approach to interpolation, and develops a new formulation of IRKA that only uses transfer function evaluations, without requiring any particular realization. This allows extensions of IRKA to $\mathcal{H}_2$ approximation of irrational, infinite-dimensional dynamical systems.

Vector fitting

The vector fitting framework (Gustavsen and Semlyen 2002; Gustavsen 2006) computes a rational approximation (no interpolation) to a set of frequency-response measurements. It does so by formulating a least-squares problem, where the sought rational approximant of the data is found as the minimizer in the L_2 sense. The optimization variables are the poles and residues of both the numerator and denominator of the barycentric form. The key is that the poles are not chosen from the sampled frequencies, but independently. Several algorithms for solving the resulting nonlinear least-squares problem exist (Drmac et al. 2015; Berljafa and Guttel 2017). While a theoretical issue with vector fitting is that convergence is not guaranteed (Lefteriu and Antoulas 2013), practitioners have observed rather robust performance (Albakri et al. 2020).

AAA algorithm

The accelerated Antoulas–Anderson (AAA) algorithm, proposed in 2018 (Nakatsukasa et al. 2018), computes rational approximations either to a specified accuracy or of a specified rational type for univariate functions on a real or complex domain, such as transfer functions for LTI systems. It has gained in popularity due to its ease of use (see the '40 lines of Matlab code' in (Nakatsukasa et al. 2018), performance and robustness. The AAA algorithm uses the barycentric formula, a quotient of two partial fractions, to represent the transfer function. The algorithm is not trying to achieve optimality with respect to the L_2 or L_∞ norm, which may be a key to its success. While the theoretical underpinnings are still studied, the model reduction community has quickly embraced this new data-driven rational interpolation algorithm as an alternative to data-driven Loewner interpolation and least-squares vector fitting. For example, extensions to parametric LTI systems yielded the p-AAA algorithm (Rodriguez et al. 2023), LTI systems with quadratic (energy-like) outputs have been considered (Gosea and Gugercin 2022), and modifications to enforce notions of system stability have been proposed in (Davis et al. 2023; Bradde et al. 2025),

3.5 Data-driven system-theoretic model reduction from time-domain measurements

A widely popular and successful method, the eigensystem realization algorithm (ERA) (Kung 1978; Juang and Pappa 1985) is representative of a large class of subspace identification techniques that work in discrete time with impulse response data. Extensions to more general data sets pair ERA with the observer Kalman filter identification algorithm (OKID), see (Juang et al. 1993). The ERA collects Markov parameters, which can be obtained by finite-differencing outputs of the system when only one input is turned on, and creates a large *Hankel matrix* of these Markov parameters. The decomposition of the Hankel matrix into the controllability and observability matrix—which can be done numerically via matrix factorizations such as SVD or CUR (Kramer and Gorodetsky 2018)—is the key step of the method. From there, the $\mathbf{B}$, $\mathbf{C}$ matrices can be read out and the $\mathbf{A}$ matrix is obtained by solving a least-squares problem.

Systems theory has also been the foundation of many other methods for data-driven model reduction. The authors in (Peherstorfer et al. 2017) extended the above-mentioned Loewner framework to work with time-domain data. For balanced truncation, balancing the system based on data-driven Gramians has been suggested for nonlinear systems in (Lall et al. 2002; Kashima 2016), as well as for LTI systems using frequency-domain snapshots (Willcox and Peraire 2002) or time-domain snapshots (Rowley 2005). These methods require solving the dynamical systems forward in time, yet no Lyapunov matrix equation has to be solved. Kernel methods have also been suggested for nonlinear balancing (Bouvrie and Hamzi 2017a, b).

4 Goal-oriented model reduction

We now shift our focus to model reduction approaches that can be described as *goal-oriented*, in the sense that the model reduction method itself is tailored to a specific intended use—the *goal*—of the ROM, in addition to the usual accuracy and runtime requirements. We find the term 'goal-oriented' useful in describing a number of model reduction approaches that do not fall neatly into either of the two major existing model reduction paradigms discussed in Sects. 3 and 2 as they have additional goals. These goals may include the prediction of specific quantities of interest or the preservation of certain geometric or physical structures in the ROM. Goal-oriented approaches may be particularly suited to digital twin applications, where the primary use of the ROM is to accelerate data assimilation and decision-making computations. However, there is certainly overlap between goal-oriented, system-theoretic, and state approximation methods: indeed, one might reasonably view the entire class of system-theoretic

model reduction methods described in Sect. 3 as being goal-oriented towards the goal of system control. Additionally, many goal-oriented approaches are closely connected with state approximation approaches, as we will discuss.

In this section we endeavor to summarize developments in these directions, with a focus on methods that have been used and are promising for engineering applications. We start in Sect. 4.1 by discussing goal-oriented snapshot selection, followed by goal-oriented basis definitions and computations in Sect. 4.2. In Sect. 4.3, we present an overview of the rich field of structure-preserving model reduction of all fashions: state approximation, systems-theory, intrusive and non-intrusive.

4.1 Goal-oriented snapshot selection

By default, state approximation methods are designed to accurately reconstruct the snapshot data used to define the low-dimensional basis. Thus, by tailoring the selection of snapshot data to the goal of interest, one may define goal-oriented versions of state approximation methods. For example, the adaptation, registration, and nonlinear transformation methods discussed in Sect. 2.1.4 target goals of preserving transport physics and translational symmetries. Another common goal is accurate prediction of a quantity of interest (rather than reconstruction of the system state). In several approaches, snapshots are collected using a greedy algorithm that targets minimizing error in the quantity of interest (Yano 2020; Sleeman and Yano 2022), sometimes in a specific goal-oriented weighted norm (Carlberg and Farhat 2011). In an optimization context, this quantity of interest may be the cost function or its derivatives (Qian et al. 2017). State derivatives may also be explicitly included in the snapshot data matrix to improve the ROM's ability to represent derivatives and improve error bounds (Carlberg and Farhat 2011; Iliescu and Wang 2014; Koc et al. 2021b). Further tailoring of the model reduction approach to the goal of optimization can be done by selecting snapshots by perturbing design variables of interest (Legresley and Alonso 2000), by including their parametric sensitivities (Hay et al. 2009), or by building the model *adaptively* during the optimization by collecting snapshots along the optimization trajectory: several works use a trust region optimization approach to do this (Qian et al. 2017; Keil et al. 2021; Wen and Zahr 2023; Yano 2020; Grundvig and Heinkenschloss 2025; Yao et al. 2020).

Adaptive snapshot selection is also used in several uncertainty quantification contexts. The works (Cui et al. 2015, 2016) develop goal-oriented ROMs for Bayesian inference by adaptively computing snapshots drawn from the posterior distribution. In (Silva et al. 2025), the authors consider the ensemble Kalman filter for data assimilation and adaptively update reduced basis models during the online filtering computations. The works (Chen and Quarteroni 2013; Heinkenschloss et al. 2020) use reduced basis error bounds to adaptively collect snapshots close to a failure region for reliability analysis and risk estimation.

4.2 Goal-oriented bases

We now discuss approaches in which the *basis* is defined not for accurate reconstruction of snapshots but in some alternative way tailored to the goal of interest. As previously alluded to, the entirety of system-theoretic methods may be viewed as belonging to this category. Beyond system-theoretic approaches, the works (Willcox et al. 2005; Bui-Thanh et al. 2007; Borggaard et al. 2016) formulate an optimization problem to select basis vectors which minimize the error in a predicted output, and show that this approach outperforms POD in output prediction accuracy. In some cases, the standard POD basis may lead to unstable models: to address this, the work (Balajewicz et al. 2016) formulates an optimization problem to minimally rotate the approximation subspace to ensure model stability. Several recent works (Qian et al. 2022; Freitag et al. 2024; König et al. 2025; Stavrinides and Qian 2025; Scheffels et al. 2025) tailor the subspace definition to the Bayesian inference setting by defining the subspace to be the directions that are maximally informed by the observed data, relative to the prior uncertainty. Finally, Peng and Mohseni (2016b) define the proper symplectic decomposition basis for structure-preserving model reduction—which we discuss in detail next.

4.3 Structure-preserving model reduction

The high-dimensional engineering models in (1) are often derived from first principles by starting from conserved quantities, as well as other physical, mechanical, and mathematical structures that are expressed in form of symmetries and symplecticity. This then leads to system invariants as a result of Noether's theorem. Thus, an additional goal of model reduction is to retain this structure in the ROM, so to guarantee meaningful and interpretable ROMs alongside long-term predictability. We will survey approaches for ROMs of Hamiltonian, Lagrangian, and port-Hamiltonian systems, whose dynamics are derived from energy (and storage) functions, e.g., the kinetic and potential energy of the system in the case of Hamiltonians, providing physical and mathematical structure. Beyond these geometric mechanics principles, other ROM goals such as preserving stability, entropy, enstrophy and other invariants have been addressed in the literature.

Lagrangian dynamical systems are derived from the Euler–Lagrange equations. Postulating that the ROM also satisfies this equation leads to Lagrangian structure-preserving ROMs, pioneered by (Lall et al. 2003). The authors in (Carl-

berg et al. 2015) expanded upon this work through several accelerations of the ROM and demonstrated their results on a clamped-free truss structure. It is necessary to preserve the structure of the Lagrangian system also during hyperreduction, as proposed in (Farhat et al. 2015) through the energy-conserving sampling and weighting (ECSW) method (Farhat et al. 2014). The authors in (Sharma et al. 2024b; Sharma and Kramer 2024; Sharma et al. 2024a) proposed fully data-driven Lagrangian Operator Inference ROMs for both linear and nonlinear models ranging from soft robotics to structural mechanics, using both simulated and experimental data. In related work, (Filanova et al. 2023) focused on second-order mechanical structure for Operator Inference learning.

Hamiltonian dynamical systems abound in many physics and engineering problems, such as Maxwell's equations, Schrödinger's equation, Korteweg–de Vries and many variants of wave equations, compressible and incompressible Euler equations, and the Vlasov–Poisson and Vlasov–Maxwell equations in plasmas. From a time-domain perspective, (Peng and Mohseni 2016b) suggested the proper symplectic decomposition to preserve the symplectic structure of the model, (Buchfink et al. 2019) provide an alternative basis computation, (Gruber and Tezaur 2025) propose a variationally consistent Hamiltonian ROM for linear Hamiltonian FOMs, and (Hesthaven and Pagliantini 2021) derive ROMs for the more general class of Poisson systems. The authors in (Maboudi Afkham and Hesthaven 2017) derive RB-ROMs for parametric Hamiltonian systems and (Gong et al. 2017; Karasözen and Uzunca 2018; Uzunca et al. 2021) propose and use a modification to Galerkin POD for Hamiltonians. Structure-preserving hyperreduction is again a key issue, as addressed in (Chaturantabut et al. 2016; Pagliantini and Vismara 2023). The accuracy of Hamiltonian RB-ROMs can be further improved by dynamically updating the basis (Pagliantini 2021). Lastly, while forcing introduces energy into a system, it is critical in all engineering applications, and extensions to forced Hamiltonian systems are considered in (Peng and Mohseni 2016a) and for other dissipative Hamiltonian systems in (Afkham et al. 2020).

Interconnected systems, such as power and gas networks, coupled sub-systems, and multi-physics problems can be modeled through the port-Hamiltonian (pH) formalism (Rashad et al. 2020), see also the up-to-date database of research in this area (PHRAISE 2025). An early survey of model reduction techniques for LTI control systems that are coupled through inputs and outputs is (Reis et al. 2008) which largely focuses on moment matching and balanced truncation. Virtually all concepts of model reduction have eventually proposed ROM solutions for the port-Hamiltonian structure: moment matching (Polyuga and van der Schaft 2010, 2011; Ionescu and Astolfi 2013), interpolatory ROMs various LTI/DAE settings (Gugercin et al. 2012; Beattie et al. 2022), generalized balanced truncation for nonlinear pH systems (Sarkar and Scherpen 2023), projection-based time-domain settings with POD/$\mathcal{H}_2$ subspaces (Beattie and Gugercin 2011; Chaturantabut et al. 2016). Moreover, there have been works focusing on a related thrust of dissipativity-preserving pH ROMs (Ionescu et al. 2010) and passivity-preserving ROMs (Wolf et al. 2010; Breiten and Unger 2022). There have also been recent successes in learning linear pH ROMs via dynamic mode decomposition (Morandin et al. 2023) or nonlinear pH ROMs via Operator Inference under certain assumptions, (Geng et al. 2025b).

Many other structure-preserving ROM approaches that do not fall nicely into the geometric mechanics classes above have found success in engineering and physics. For example, researchers have been able to conserve first integrals (Miyatake 2019), power-balance (Balajewicz et al. 2013), and energy (Sanderse 2020) in Navier–Stokes equations, skew-symmetry for hyperbolic systems (Afkham et al. 2020), and energy, enstrophy, mass, circulation in shallow water equations (Karasözen et al. 2021). When the high-dimensional model is a finite-volume model, the authors in (Carlberg et al. 2018) suggest a conservative model reduction approach in the discrete-time (least-squares Petrov–Galerkin) and continuous-time (Petrov–Galerkin) setting. The work (Beattie and Gugercin 2009) proposes a system-theoretic framework for LTI systems based on normalized co-prime factorization that can preserve symmetries, second- and higher-order structure, state constraints, internal delays, and infinite-dimensional sub-systems.

Stability is a central goal for ROMs, and while many of the approaches above guarantee stability directly or indirectly through their structure preservation, other ROMs—specifically those derived through Galerkin projection—may require additional steps to ensure this goal. In fact, the LSPG ROM developments discussed above were driven in part by instabilities in standard Galerkin ROMs. In another research direction, initially unstable ROMs can be made stable via eigenvalue reassignment (Kalashnikova et al. 2014), specific regularization of the Operator Inference ROM-learning problem can create stable models (Sawant et al. 2023), leveraging the trapping theorem can provide stability certificates for quadratic-bilinear systems (Peng et al. 2025), and entropy stability of reduced basis ROMs can be achieved following (Chan 2020).

5 Application highlights: ROMs for engineering systems

Digital twins will provide the greatest value for systems of high complexity and where decisions and actions have high consequences. The digital twin allows one to hypothesize 'what-if' scenarios that are experimentally not possible, as

well as to optimize and control complex engineered systems. Model reduction has demonstrated many successes over the past two decades in delivering ROMs with orders of magnitude reduced simulation costs, while maintaining the accuracy required for a specific task. In this section, we highlight a few—but by no means all—success stories. While many of the high-dimensional problems naturally arise from fluid mechanics, see (Rozza et al. 2022) for a recent review of advanced ROMs in fluid mechanics, we highlight a variety of applications to give a flavor of the broad applicability of reduced-order modeling.

The authors in (Lieu et al. 2006) generated a ROM for a complete F-16 aircraft configuration, and (Kapteyn et al. 2022) built a digital twin for the physical twin of a 12ft wingspan unmanned aerial vehicle. The latter work started with component-based ROMs of the pristine aircraft's structure and updated the model as the vehicle was flown and collected sensor data. Continuum soft robots provided another area where ROMs were instrumental for real-time control of underactuated systems (Goury and Duriez 2018; Sharma et al. 2024a). In (Carlberg et al. 2013), the authors considered the turbulent flow over the Ahmed body and reduced the 17 million degrees of freedom model to a handful. Reactive flow simulations have seen some of the highest cost reductions through ROMs to date due to their coupled multiphysics behavior and extreme cost of high-fidelity solution. For the problem of a multi-injector two-dimensional rocket engine, (Huang et al. 2022) propose a strategy to divide physically large models into multiple components, then simulate individual ROMs, couple the individual ROMs together into a global ROM. The authors in (Qian et al. 2022) considered three-dimensional rocket combustion simulation with over 18 million degrees of freedom, and through Operator Inference-based ROMs, the authors achieved runtime reduction of up to nine orders of magnitude.

The work (Blonigan et al. 2021) proposes least-squares Petrov–Galerkin model reduction for a hypersonic three-dimensional, turbulent Mach 7.1 flow, where the full-order model has over 12 million degrees of freedom. Moreover, a series of ROM works (Farcaş et al. 2023, 2024, 2025) focused on three-dimensional rotating detonation engines, where the high-fidelity models require more than 10,000 cores and 5–10 million CPU-hours to simulate 1–2 ms of physical time. The original models had over 75 million degrees of freedom, and runtime reduction was shown to be over six orders of magnitude. In a plasma application, the authors in (Gahr et al. 2024) built two-dimensional electrostatic drift-wave turbulence ROMs.

6 Software and benchmarks

For engineers and practitioners looking to use reduced-order models for their specific goals and digital twin development, there are several software packages that are currently actively maintained, have user groups, and have workshops and conferences centered around their use. First, one of the most successful early contributors was SLICOT (SLICOT 2017; Benner et al. 1999), which provides Fortran 77 implementations that formed the bases for many Matlab numerical algorithms for computations in systems and control theory. Second, a more recent Python-based model reduction software library is pyMOR (Milk et al. 2016; pyMOR 2025). All algorithms in pyMOR are formulated in terms of abstract interfaces, allowing generic implementations to work with different back-ends, from NumPy/SciPy to external partial differential equation solver packages. Third, there is the RBniCS Project (Rozza et al. 2024; RBniCS 2025), a FEniCS-based reduced-order modeling package with a particular focus on the certified reduced basis method and proper orthogonal decomposition-Galerkin methods for parametrized systems. Fourth, the Operator Inference python package (OPINF 2025) offers a variety of solutions and numerical methods to efficiently learn polynomial reduced-order models from data. Fifth, the Matrix Equation Sparse Solver (M.E.S.S.) software (Saak et al. 2025) is a useful tool in system-theoretic model reduction, where high-dimensional matrix equations have to be solved. Lastly, there are reduced-order modeling software packages, including several test problems, by Sandia National Laboratories (Pressio (Rizzi et al. 2021)) and Lawrence Livermore National Laboratories (libROM (Choi et al. 2019)).

The model reduction community has developed several databases and repositories where new models can be tested and benchmarked against other techniques. For instance, the Oberwolfach benchmark collection (Korvink et al. 2005) was one of the earliest attempts at having a large collection of models of different characteristics. This collection is included in the MOR Wiki project (The MORwiki Community 2025), which includes many more benchmarks (approximately 70 models) of parametric or non-parametric models and also material explaining applicable (parametric) model reduction methods as well as available software implementations. The work (Park et al. 2024) resulted from a joint effort between seven model reduction groups worldwide, where seven state-of-the-art ROM methods were evaluated and compared in terms of accuracy and efficiency in capturing the nonlinear characteristics of a curved, perforated plate that is part of the exhaust system of a large diesel engine.

7 Conclusion and opportunities for digital twins

This survey is intended to provide the reader with an overview and understanding of the rich set of tools and theories developed in model reduction over the past half-century. As with any such writing effort, trade-offs must be made between the accessibility of the exposition and the level of detail. We have generally chosen to focus on providing higher-level descriptions of the methods, recognizing that this approach cannot do justice to many contributions throughout five successful decades of model reduction research.

We now conclude with some discussion of the opportunities and challenges for model reduction in the context of digital twins. As discussed previously, the success of digital twins will rely critically on two types of computations: (1) physical-to-digital (P2D) synchronization of the digital model based on up-to-date data acquired from the physical system, and (2) digital-to-physical (D2P) use of the digital model for real-time control, operation, and maintenance decision-making for the physical system. In both directions, surrogate models that are orders of magnitude cheaper than high-fidelity full-order models are required to make real-time computations tractable. The success of projection-based ROMs in cheaply approximating complex nonlinear systems with tens of millions of unknowns (cf. Sect. 5) demonstrates the potential of ROMs as an enabling technology for digital twins. However, there remain a number of open challenges in the integration of ROMs into digital twins.

First, many complex physical systems with high inherent dimensionality continue to pose barriers to efficient dimension reduction. Problems with high Kolmogorov n-width, including transport-dominated problems arising in fluids, form one class of such problems. In the D2P digital twin decision-making context, the high-dimensional control and decision spaces can pose challenges to efficient reduction, as can high parameter dimensions in the P2D parameter updating context. These challenges are the focus of continued active research within the mathematical model reduction research communities, where localized, adaptive, and machine learning approaches have shown early promise.

A second challenge in model reduction for digital twins is the system integration required for digital twins, which creates difficulties at multiple levels. At the discipline level, many ROMs are developed for single-physics systems (e.g. structural only or fluid only) and accurate modeling of multi-physics systems requires careful coupling between models. At the decision-making level, ROMs are often developed primarily for single goals such as state approximation (cf. Sect. 2) *or* control (cf. Sect. 3); however a ROM within a digital twin must often simultaneously serve multiple purposes ranging from inference to control to optimization. While strategies for multi-physics coupling have garnered some attention, more remains to be done, and little attention has been devoted to the challenge of '*multiple* goal-oriented' ROM development.

A final critical challenge is that of verification, validation, and uncertainty quantification (VVUQ) of ROMs for digital twins, which is critical to establishing trust in their use for decision-making and exploring "what-if" scenarios for physical assets. Approximation of very high-dimensional systems with low-dimensional models will inevitably incur approximation errors in all but the simplest of applications; these errors in the ROM will then propagate to errors in both P2D and D2P computations. While rigorous mathematical error bounds are available for some types of ROMs in some settings (often linear or only weakly nonlinear systems), such theoretical guarantees are usually not available for strongly nonlinear models which appear in many compelling use cases for digital twins. In such cases, alternative error heuristics must be developed, for example, existing approaches may use the magnitude of the residual of the governing equations (Zahr and Farhat 2015) or a machine learning model of the error as error indicators (Lipponen et al. 2018). These approaches are highly problem-dependent and significant opportunity for further research remains in these directions. Alternatively, outer-loop computations for digital twins (e.g., optimization, UQ, and data assimilation) can be conducted in error-aware ways, for example, by using robust optimization approaches that account for model error in evaluation of the cost function and constraints, or by using multifidelity strategies that leverage reduced models for computational acceleration while retaining limited evaluations of the high-fidelity model to guarantee accuracy (Peherstorfer et al. 2018). Additionally, further research into VVUQ in systems of integrated models is required to address the compounding challenges posed by the system integration setting discussed above.

To conclude: research in projection-based model reduction methods has advanced significantly over the last five decades, and now provides a reliable suite of tools for computationally efficient approximation of high-dimensional engineering models across a range of physical systems and model use cases. There is therefore significant promise for the use of ROMs in digital twins for engineering systems, yet considerable challenges remain. To realize the promise of ROMs for the digital twins, further research will be required, particularly in close collaboration between domain and systems engineering researchers and researchers with expertise in the computational and mathematical aspects of reduced-order modeling.

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